Konsep Integral

47.2 The general solution of integrals of the form ax^n

The general solution of integrals of the form $\int ax^n dx$, where a and n are constants is given by:

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$$

This rule is true when n is fractional, zero, or a positive or negative integer, with the exception of $n = -1$.

Konsep Integral

Using this rule gives:

(i) $$\int 3x^4 \, dx = \frac{3x^{4+1}}{4+1} + c = \frac{3}{5}x^5 + c$$

(ii) $$\int \frac{2}{x^2} \, dx = \int 2x^{-2} \, dx = \frac{2x^{-2+1}}{-2+1} + c$$

$$= \frac{2x^{-1}}{-1} + c = \frac{-2}{x} + c, \text{ and}$$

Konsep Integral

(a) The integral of a constant k is $kx + c$. For example,

$$\int 8\,dx = 8x + c$$

(b) When a sum of several terms is integrated the result is the sum of the integrals of the separate terms. For example,

$$\int (3x + 2x^2 - 5)\,dx$$

$$= \int 3x\,dx + \int 2x^2\,dx - \int 5\,dx$$

$$= \frac{3x^2}{2} + \frac{2x^3}{3} - 5x + c$$

Konsep Integral

47.4 Definite integrals

Integrals containing an arbitrary constant c in their results are called **indefinite integrals** since their precise value cannot be determined without further information. **Definite integrals** are those in which limits are applied. If an expression is written as $[x]_a^b$, 'b' is called the upper limit and 'a' the lower limit.

The operation of applying the limits is defined as: $[x]_a^b = (b) - (a)$

Konsep Integral

$$\int_1^3 x^2\,dx = \left[\frac{x^3}{3} + c\right]_1^3 = \left(\frac{3^3}{3} + c\right) - \left(\frac{1^3}{3} + c\right)$$
$$= (9 + c) - \left(\frac{1}{3} + c\right) = \mathbf{8\frac{2}{3}}$$

$$\int_1^2 3x\,dx = \left[\frac{3x^2}{2}\right]_1^2 = \left\{\frac{3}{2}(2)^2\right\} - \left\{\frac{3}{2}(1)^2\right\}$$
$$= 6 - 1\frac{1}{2} = \mathbf{4\frac{1}{2}}$$

Variabel Random Kontinu

- Pengukuran-2 berat badan, panjang, diameter, dsb, dinyatakan dg variabel kontinu. Variabel kontinu menyatakan sembarang nilai dalam suatu interval. Fungsinya dinamakan Fungsi Kepadatan Probabilitas (*probability density function*). Identik Luasan.
- Jika X merupakan variabel random kontinu yg bernilai - ~ s/d + ~ maka Fungsi Kepadatan yang menggambarkan probabilitas X dalam interval a s/d b dimana a ≤ b ialah :

$$f(x) = p(a < X < b\,) = \int_a^b f(x)\,dx$$

V.R. Kontinu – Contoh #1

Variabel random X memiliki Fungsi Kepadatan sbb. :

- $f(x) = 0 \; ; x \leq 2$
- $f(x) = \frac{1}{18}.(3 + 2.x) \; ; 2 < x < 4$
- $f(x) = 0 \; ; x \geq 4$

Jika x = 2 dan x’ = 3, berapakah p(x < X < x’) atau berapakah p (a < X < b) ?

Jawab :

$$p(x < X < x') = \int_2^3 \frac{1}{18}.(3 + 2.x)dx = \frac{4}{9}$$

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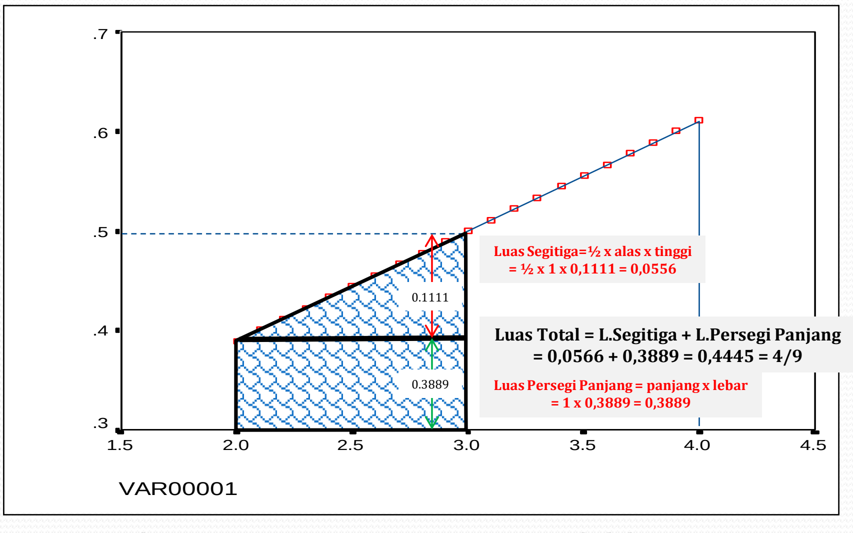
V.R. Kontinu – Contoh #1

$$f(x) = \frac{1}{18}.(3 + 2.x)$$

no	x	f(x)
1	2,00	0,3889
2	2,10	0,4000
3	2,20	0,4111
4	2,30	0,4222
5	2,40	0,4333
6	2,50	0,4444
7	2,60	0,4556
8	2,70	0,4667
9	2,80	0,4778
10	2,90	0,4889
11	3,00	0,5000
12	3,10	0,5111
13	3,20	0,5222
14	3,30	0,5333
15	3,40	0,5444
16	3,50	0,5556
17	3,60	0,5667
18	3,70	0,5778
19	3,80	0,5889
20	3,90	0,6000
21	4,00	0,6111



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V.R. Kontinu – Contoh #2

Variabel random X memiliki Fungsi Kepadatan sbb. :

- $f(x) = 2.x\,; 0 < x < 1$
- $f(x) = 0\,; lainnya.$

a. Berapakah $p(\frac{1}{2} < X < \frac{3}{4})$?

b. Berapakah $p(-\frac{1}{2} < X < \frac{1}{2})$?

Jawab :

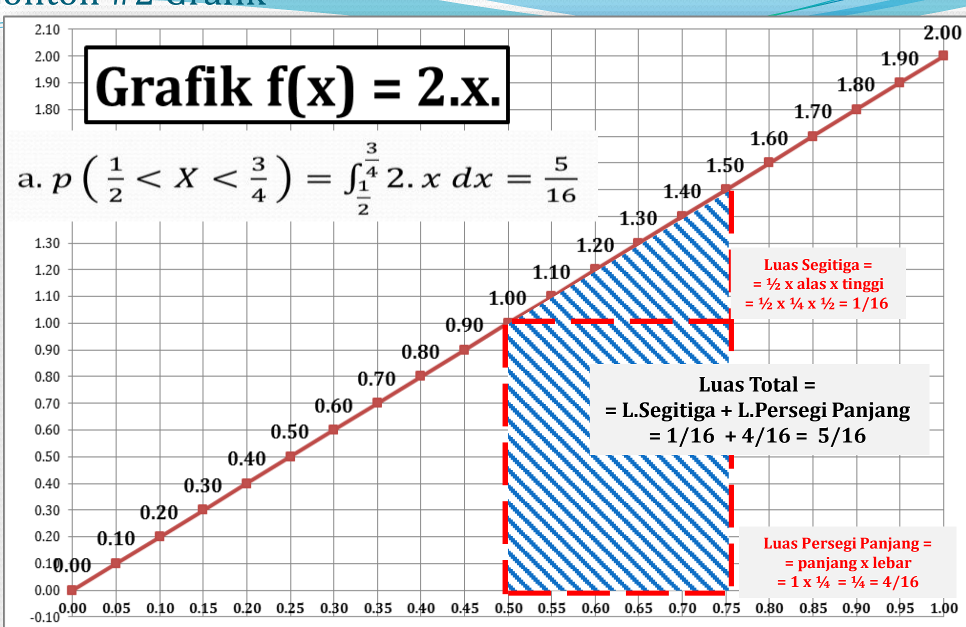
a. $p\left(\frac{1}{2} < X < \frac{3}{4}\right) = \int_{\frac{1}{2}}^{\frac{3}{4}} 2.x\,dx = \frac{5}{16}$

b. $p\left(-\frac{1}{2} < X < \frac{1}{2}\right) = \int_{-\frac{1}{2}}^{\frac{1}{2}} 2.x\,dx = \int_{-\frac{1}{2}}^{0} 2.x\,dx + \int_{0}^{\frac{1}{2}} 2.x\,dx = 0 + \frac{1}{4} = \frac{1}{4}$

V.R. Kontinu – Contoh #2 Grafik

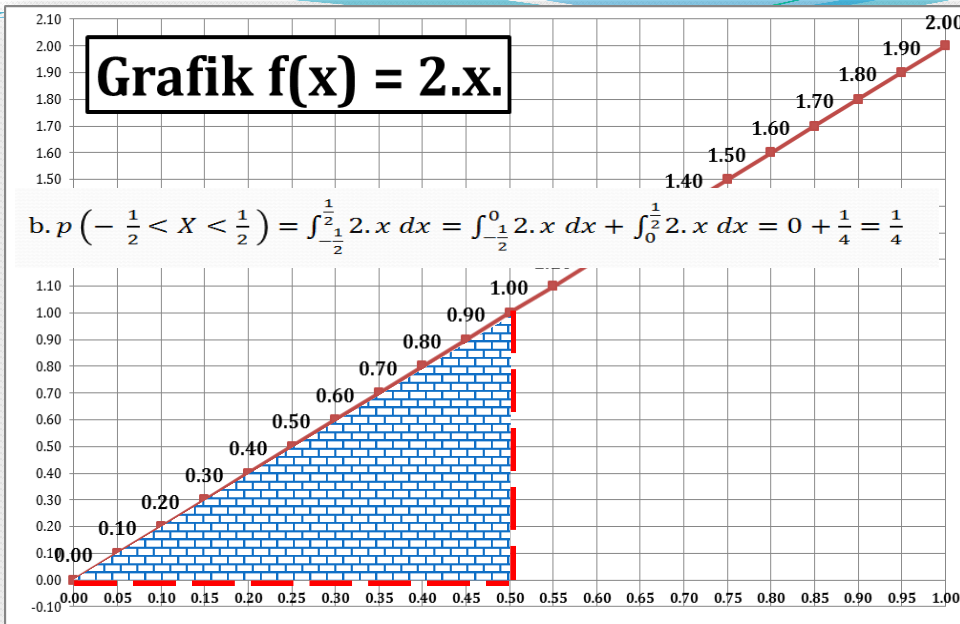
f(x) = 2.x.

No	x	f(x)
1	0.00	0.00
2	0.05	0.10
3	0.10	0.20
4	0.15	0.30
5	0.20	0.40
6	0.25	0.50
7	0.30	0.60
8	0.35	0.70
9	0.40	0.80
10	0.45	0.90
11	0.50	1.00
12	0.55	1.10
13	0.60	1.20
14	0.65	1.30
15	0.70	1.40
16	0.75	1.50
17	0.80	1.60
18	0.85	1.70
19	0.90	1.80
20	0.95	1.90
21	1.00	2.00



V.R. Kontinu – Contoh #2 Grafik

f(x) = 2.x.		
No	x	f(x)
1	0.00	0.00
2	0.05	0.10
3	0.10	0.20
4	0.15	0.30
5	0.20	0.40
6	0.25	0.50
7	0.30	0.60
8	0.35	0.70
9	0.40	0.80
10	0.45	0.90
11	0.50	1.00
12	0.55	1.10
13	0.60	1.20
14	0.65	1.30
15	0.70	1.40
16	0.75	1.50
17	0.80	1.60
18	0.85	1.70
19	0.90	1.80
20	0.95	1.90
21	1.00	2.00



b. $p\left(-\frac{1}{2} < X < \frac{1}{2}\right) = \int_{-\frac{1}{2}}^{\frac{1}{2}} 2.x\ dx = \int_{-\frac{1}{2}}^{0} 2.x\ dx + \int_{0}^{\frac{1}{2}} 2.x\ dx = 0 + \frac{1}{4} = \frac{1}{4}$

VRK – Rata-rata & Standart Deviasi

Rata-rata : $\mu = \int_a^b x.f(x)\ dx$

Variansi :

#1 #2

$\sigma_x^2 = \int_a^b (x-\mu)^2.f(x)\ dx = \int_a^b x^2.f(x)\ dx - \mu^2$

Standart Deviasi : $\sigma_x = \sqrt{\sigma_x^2}$

VRK – Rata-rata & Standart Deviasi

Variabel random X memiliki fungsi kepadatan probabilitas sbb. :

- $f(x) = 0\ ; x \leq 0$
- $f(x) = \frac{3}{8}.(x-2)^2\ ;\ 0 < x < 2$
- $f(x) = 0\ ; x \geq 2$

Tentukan Rata-rata, Varians & Standart Deviasi-nya !

Rata-rata =

$$\mu = \int_a^b x.f(x)\,dx = \int_0^2 x.\frac{3}{8}.(x-2)^2\,dx = \int_0^2 x.\frac{3}{8}.(x^2-4.x+4)dx$$

$$\mu = \frac{3}{8}\int_0^2 x.(x^2-4.x+4)dx = \frac{3}{8}\int_0^2 (x^3-4.x^2+4.x)\,dx = \frac{3}{8}.\left[\frac{1}{4}.x^4 - 4.\frac{1}{3}.x^3 + 4.\frac{1}{2}.x^2\right]_0^2$$

$$\mu = \frac{3}{8}.\left[\left[\frac{1}{4}.2^4 - 4.\frac{1}{3}.2^3 + 4.\frac{1}{2}.2^2\right] - \left[\frac{1}{4}.0^4 - 4.\frac{1}{3}.0^3 + 4.\frac{1}{2}.0^2\right]\right]$$

$$\mu = \frac{3}{8}.\left[\left[\frac{1}{4}.16 - 4.\frac{1}{3}.8 + 4.\frac{1}{2}.4\right] - \left[\frac{1}{4}.0^4 - 4.\frac{1}{3}.0^3 + 4.\frac{1}{2}.0^2\right]\right]$$

$$\mu = \frac{3}{8}.\left[\left[\frac{16}{4} - \frac{32}{3} + \frac{16}{2}\right] - [0]\right] = \frac{3}{8}.\left[4 - \frac{32}{3} + 8\right] = \frac{3}{8}.\left[12 - 10\frac{2}{3}\right] = \frac{3}{8}.1\frac{1}{3} = \frac{3}{8}.\frac{4}{3} = \frac{4}{8} = \frac{1}{2}$$

Variansi #1 :

$$\sigma_x^2 = \int_a^b (x-\mu)^2 . f(x)\,dx = \int_0^2 (x-\frac{1}{2})^2 . \frac{3}{8} . (x-2)^2\,dx$$

$$\sigma_x^2 = \frac{3}{8} . \int_0^2 (x-\frac{1}{2})^2 . (x^2 - 4.x + 4)\,dx = \frac{3}{8} . \int_0^2 \left(x^2 - x + \frac{1}{4}\right)(x^2 - 4.x + 4)\,dx$$

$$\sigma_x^2 = \frac{3}{8} . \int_0^2 x^4 - 4.x^3 + 4.x^2 - x^3 + 4.x^2 - 4.x + \frac{1}{4}.x^2 - x + 1\,dx$$

$$\sigma_x^2 = \frac{3}{8} . \int_0^2 x^4 - 5.x^3 + 8\frac{1}{4}.x^2 - 5.x + 1\,dx = \frac{3}{8} . \left[\frac{1}{5}.x^5 - 5.\frac{1}{4}.x^4 + 8.\frac{1}{4}.\frac{1}{3}.x^3 - 5.\frac{1}{2}.x^2 + x\right]_0^2$$

$$\sigma_x^2 = \frac{3}{8} . \left[\left[\frac{1}{5}.2^5 - 5.\frac{1}{4}.2^4 + 8.\frac{1}{4}.\frac{1}{3}.2^3 - 5.\frac{1}{2}.2^2 + 2\right] - \left[\frac{1}{5}.0^5 - 5.\frac{1}{4}.0^4 + 8.\frac{1}{4}.\frac{1}{3}.0^3 - 5.\frac{1}{2}.0^2 + 0\right]\right]$$

$$\sigma_x^2 = \frac{3}{8} . \left[\left[\frac{32}{5} - 5.\frac{16}{4} + \frac{33}{4}.\frac{8}{3} - \frac{20}{2} + 2\right] - [0]\right] = \frac{3}{8} . \left[6\frac{2}{5} - 20 + 22 - 10 + 2\right]$$

$$\sigma_x^2 = \frac{3}{8} . \left[6\frac{2}{5} + 22 + 2 - 20 - 10\right] = \frac{3}{8} . \left[30\frac{2}{5} - 30\right] = \frac{3}{8} . \frac{2}{5} = \frac{3}{20} = 0{,}15$$

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Variansi #2 :

$$\sigma_x^2 = \int_a^b x^2 . f(x)\,dx - \mu^2$$

$$\int_a^b x^2 . f(x)\,dx = \int_0^2 x^2 . \frac{3}{8} . (x-2)^2\,dx = \frac{3}{8}\int_0^2 x^2 . (x^2 - 4.x + 4)\,dx = \frac{2}{5}$$

$$\int_a^b x^2 . f(x)\,dx = \frac{3}{8}\int_0^2 x^4 - 4.x^3 + 4x^2\,dx = \frac{3}{8} . \left[\frac{1}{5}.x^5 - 4.\frac{1}{4}.x^4 + 4.\frac{1}{3}.x^3\right]_0^2$$

$$\int_a^b x^2 . f(x)\,dx = \frac{3}{8} . \left[\left[\frac{1}{5}.32 - 16 + \frac{4}{3}.8\right] - [0]\right] = \frac{3}{8} . \left[\frac{32}{5} - 16 + \frac{32}{3}\right] = \frac{3}{8} . \left[6\frac{2}{5} - 16 + 10\frac{2}{3}\right]$$

$$\int_a^b x^2 . f(x)\,dx = \frac{3}{8} . \left[6\frac{2}{5} + 10\frac{2}{3} - 16\right] = \frac{3}{8} . \left[16\frac{6+10}{15} - 16\right] = \frac{3}{8} . \left[16\frac{6+10}{15} - 16\right]$$

$$\int_a^b x^2 . f(x)\,dx = \frac{3}{8} . \left[\frac{16}{15}\right] = \frac{3}{8} . \frac{16}{15} = \frac{2}{5}$$

Sehingga Varians #2:

$$\sigma_x^2 = \int_a^b x^2 . f(x)\,dx - \mu^2 = \frac{2}{5} - \left(\frac{1}{2}\right)^2 = \frac{2}{5} - \frac{1}{4} = \frac{8}{20} - \frac{5}{20} = \frac{3}{20} = 0{,}15$$

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VRK – Rata-rata & Standart Deviasi

Jawab :

Standart Deviasi :

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{0,15} = 0,3873$$

Sekian